

Edge Detection ( Turunan ke dua )

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# Using Second Derivatives for Image Enhancement

- The 2<sup>nd</sup> derivative is more useful for image enhancement than the 1<sup>st</sup> derivative
  - Stronger response to fine detail
  - Simpler implementation
  
- The first sharpening filter we will look at is the *Laplacian*
  - Isotropic
  - One of the simplest sharpening filters
  - We will look at a digital implementation

# The Laplacian

- The Laplacian is defined as follows:

$$\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$$

- where the partial 1<sup>st</sup> order derivative in the  $x$  direction is defined as follows:

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

- and in the  $y$  direction as follows:

$$\frac{\partial^2 f}{\partial^2 y} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

## Laplacien Operator (seconde derivative)

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$f(x, y) = xy^3 + \sin(x) \quad \Rightarrow \quad \nabla^2 f = -\sin(x) + 6xy$$

$$\begin{array}{c} /x \downarrow \qquad \qquad \downarrow /y \\ \nabla f = [y^3 + \cos(x) \quad 3xy^2] \end{array}$$

$$\nabla f = [y^3 + \cos(x) \quad 3xy^2]$$

$$\begin{array}{l} \xrightarrow{/x} \\ \xrightarrow{/y} \end{array}$$

$$\begin{bmatrix} -\sin(x) & 3y^2 \\ 3y^2 & 6xy \end{bmatrix} = Hf$$

/y

## Laplacien operator

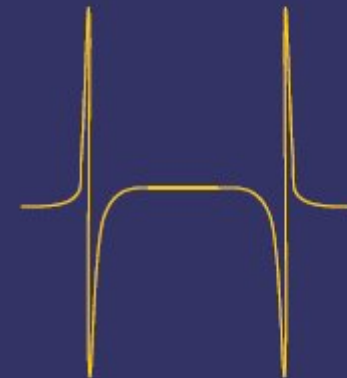
0	-1	0
-1	4	-1
0	-1	0

● Noise sensitive

● Double ligne generation

● 0 crossing

$$R = \sum w_i f_i = 4f_5 - (f_2 + f_4 + f_6 + f_8)$$



# The Laplacian (cont...)

- So, the Laplacian can be given as follows:

$$\begin{aligned}\nabla^2 f = & [f(x+1, y) + f(x-1, y) \\ & + f(x, y+1) + f(x, y-1)] \\ & - 4f(x, y)\end{aligned}$$

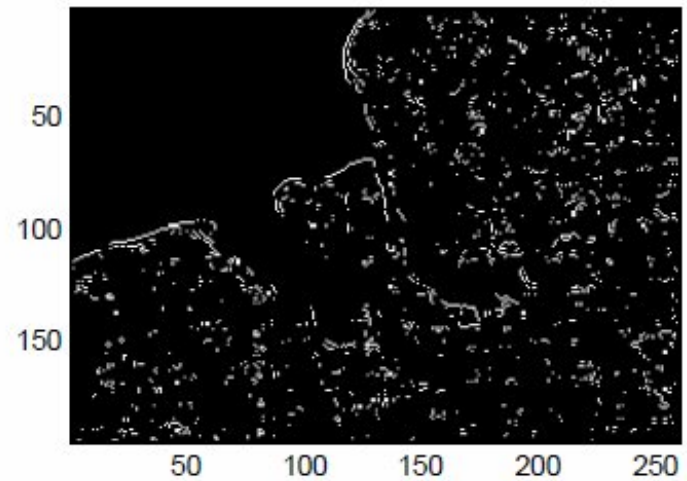
- We can easily build a filter based on this

0	1	0
1	-4	1
0	1	0

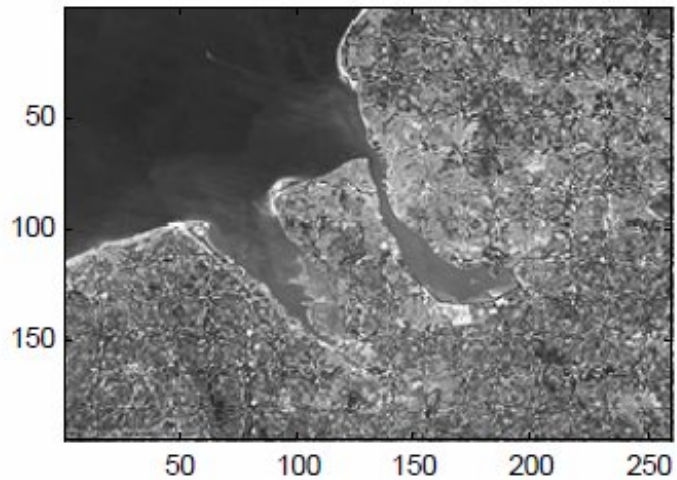
Liverpool RGB



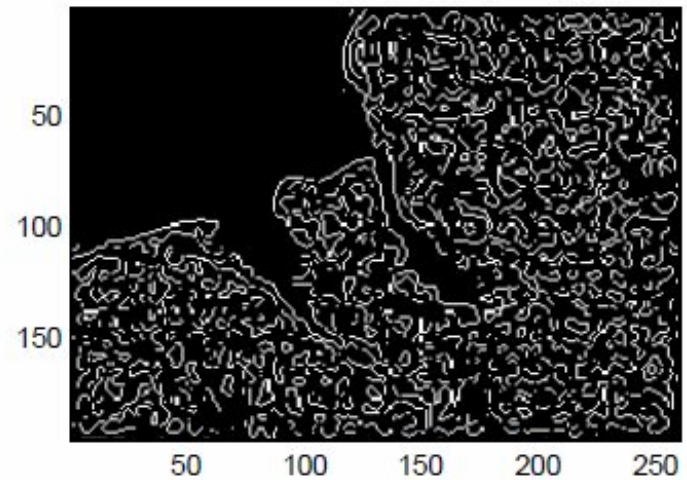
Sobel



Liverpool 256 gris

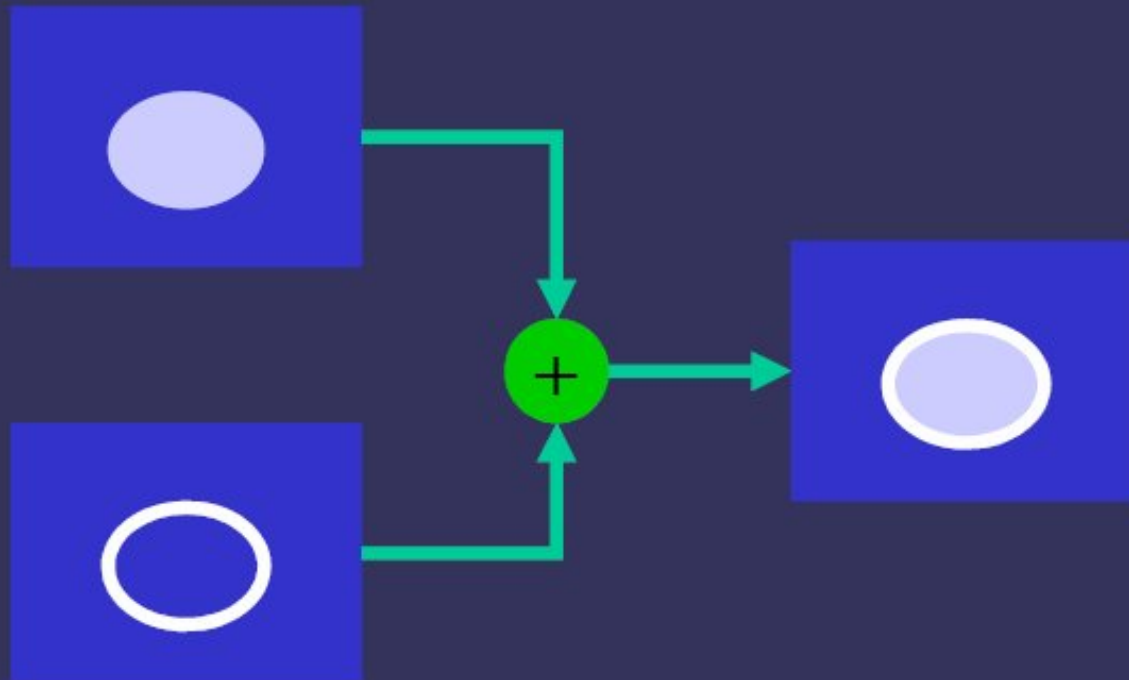


Laplacien



## Spatial Filtering

Edges detected by the Laplacian can be used to sharpen the image !



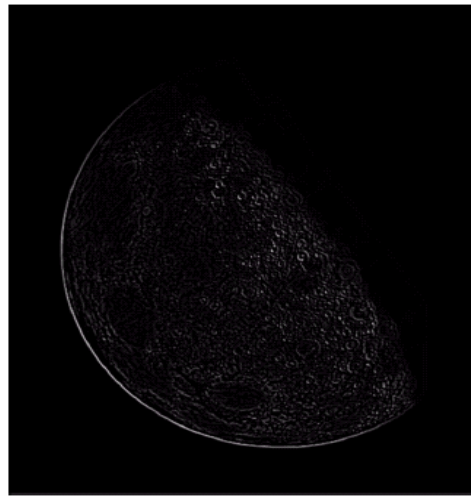


# The Laplacian (cont...)

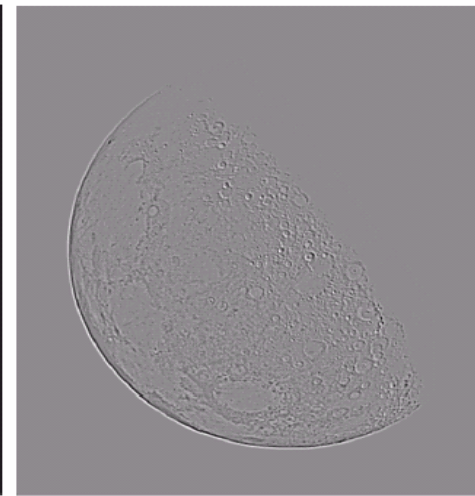
- Applying the Laplacian to an image we get a new image that highlights edges and other discontinuities



Original  
Image



Laplacian  
Filtered Image



Laplacian  
Filtered Image  
Scaled for Display

# But That Is Not Very Enhanced!

- The result of a Laplacian filtering is not an enhanced image
- We have to do more work in order to get our final image
- Subtract the Laplacian result from the original image to generate our final sharpened enhanced image

$$g(x, y) = f(x, y) - \nabla^2 f$$



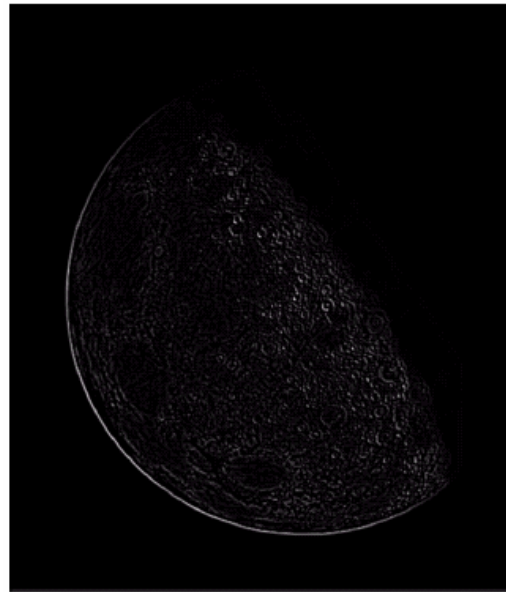
Laplacian  
Filtered Image  
Scaled for Display

# Laplacian Image Enhancement



Original  
Image

-



Laplacian  
Filtered Image

=



Sharpened  
Image

- In the final sharpened image edges and fine detail are much more obvious

# Simplified Image Enhancement

- The entire enhancement can be combined into a single filtering operation

$$\begin{aligned}g(x, y) &= f(x, y) - \nabla^2 f \\ &= f(x, y) - [f(x+1, y)f(x-1, y) \\ &\quad + f(x, y+1) + f(x, y-1)] \\ &\quad + 4f(x, y) \\ &= 5f(x, y) - [f(x+1, y)f(x-1, y) \\ &\quad + f(x, y+1) + f(x, y-1)]\end{aligned}$$

Sharpening can be done in 1 pass:

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

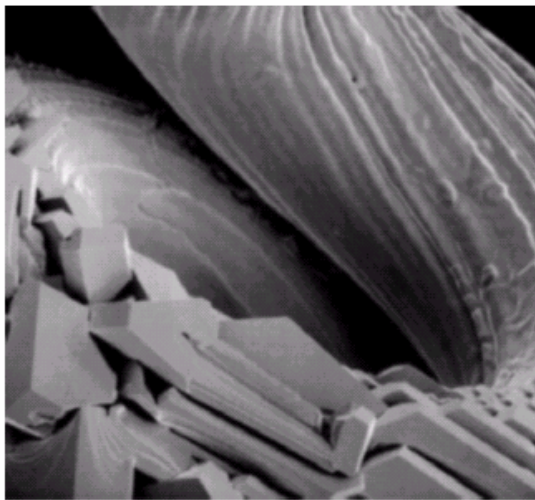
LAPLACIAN

Original Image

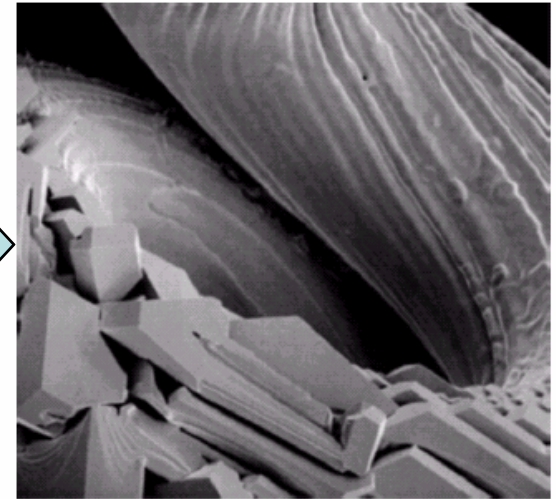
Sharpened Image

# Simplified Image Enhancement (cont...)

- This gives us a new filter which does the whole job for us in one step



0	1	0
1	5	1
0	1	0



# Variants on the Simple Laplacian

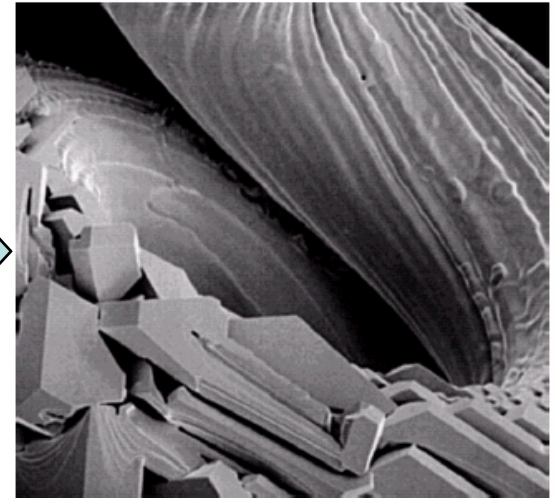
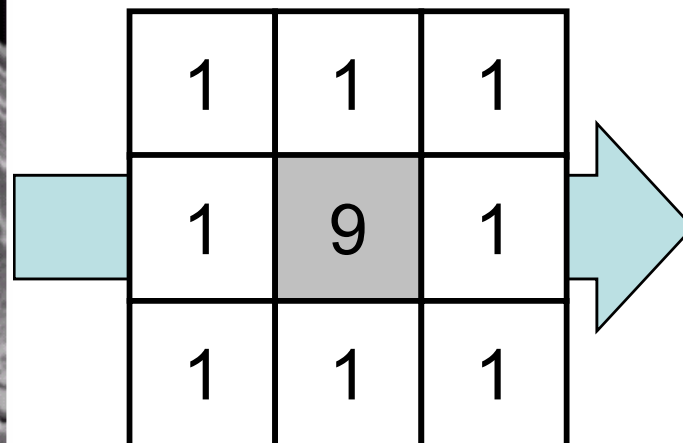
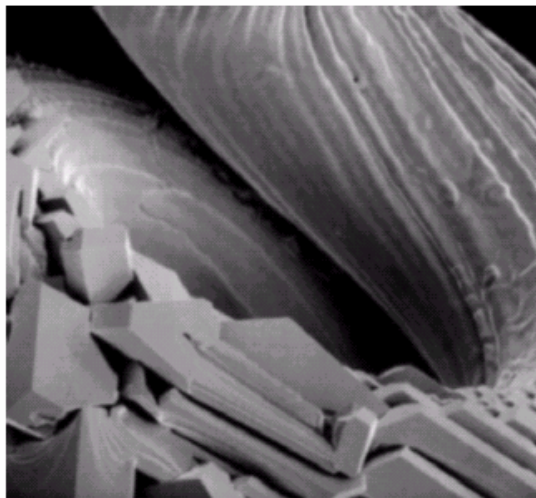
- There are lots of slightly different versions of the Laplacian that can be used:

0	1	0
1	-4	1
0	1	0

Simple  
Laplacian

1	1	1
1	-8	1
1	1	1

Variant of  
Laplacian





## Spatial Filtering

### Effect of the Laplacian... (MATLAB demo)

```
im = im/max(im(:));  
  
% create laplacian  
L=[1 1 1;1 -8 1; 1 1 1];  
  
% Filter !  
im1=conv2(im,L);  
  
% normalize and show  
im1=(im1-min(im1(:))) / (max(im1(:))-min(im1(:)));  
imshow(im1);
```





## Spatial Filtering

